Parallel Possibility Results of Preference Aggregation and Strategy-Proofness by Using Prolog

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Objectives

- ✓ According to Arrow's impossibility theorem, rational collective decision-making should be dictatorial under certain moderate assumptions. Similarly, if a voting procedure is strategy-proof (i.e., nonmanipulable), then it is dictatorial (Gibbard-Satterthwaite theorem). In these classical studies, agent's rankings are unrestricted.
- ✓ This paper presents the exact numbers of Arrow-type preference aggregation rules (SWFs) and Gibbard-Satterthwaite-type strategy-proof voting procedures (SCFs) for 2-person 3-alternative linear preference ordering (i.e., the base case) under restricted domains.

1: a c b
2: a b c
3: b a c
4: b c a
5: c b a
6: c a b

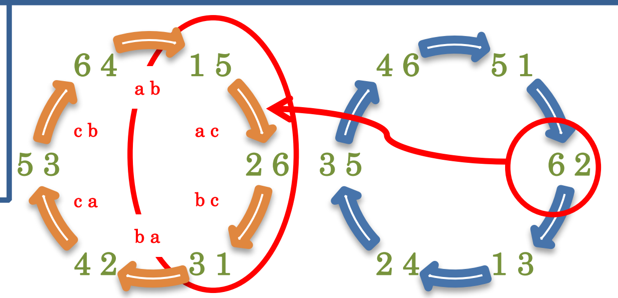

Fig. Two minimal super-Arrovian domains for the base case, cross adjacent profile pairs for a profile 62. Two cycles propagates the decisiveness of Agent 1 (left) and of Agent 2 (right) for each *xy*. Switching directions of the arrows (and *xy* to *yx*) changes the dictator.

A parallel possibility where 4 (resp. b) for the SWF (resp. SCF) is chosen unless both agents can agree.

S W F
123456

1: 123456
2: 22344-
3: 333444
4: 444444
5: 544455
6: 6-4456

S C F
123456

1: aabbcc
2: aabbb-
3: bbbbbb
4: bbbbbb
5: cbbbcc
6: c-bbcc

Methods

- ✓ A subset of profiles which suffice to prove a dictatorship is called *super-Arrovian domain* [1]. There are two such sets each of which consists of six profiles (See above figure).
- ✓ Nondictatorial SWFs and SCFs can be generated by removing (a part of) these twelve profiles.
- ✓ We adopt Prolog for modeling the social choice [2][3][4].

Model

- ✓ A ranking is complete, transitive, asymmetric ordering.
- ✓ A collective choice is defined as a function over profiles.
- ✓ An SWF satisfies transitivity, unanimity, and independence.
- ✓ An SCF satisfies transitivity, non-imposition, and strategy-proof.
- ✓ An SCF is strategy-proof if no agent ever benefits from misreporting on his/her ranking.

Results

The findings of the presented paper can be summarized into the following three results and three tables.

Table 1: Arrow-type preference aggregation.

Number of profiles remaining in the minimal super-Arrovian domains

Number of SWFs including two dictatorships (top row indicates impossibility results).

Number of restricted domains on which an SWF exists.

#swf	0	1	2	3	4	5	6	7	8	9	10	11	12	total
2							2	12	48	76	48	12	1	199
3								60	156	108	18			342
4							54	228	225	36				543
5						12	170	348	60					590
6						60	390	120	6					576
7						228	252	24						504
8					48	348	50							446
9					156	120	6							282
10					225	24								249
11				76	60									136
12				108	6									114
13				36										36
14			48											48
15			18											18
17		12												12
20	1													1
total	1	12	66	220	495	792	924	792	495	220	66	12	1	4096

Conclusions

Logic programming can be beneficial to computational study of the axiomatic collective decision-making and mechanism design, not only for automatic proving well-known theorems but also for exploring (i.e., dada-mining) new results.

Table 2: Non-imposed strategy-proof voting procedures (SCFs).

#scf	0	1	2	3	4	5	6	7	8	9	10	11	12	total
2							2	12	30	64	48	12	1	169
3									114	120	18			252
4								144	255	36				435
5							62	300	90					452
6						12	150	252	6					420
7							294	72						366
8						120	242	12						374
9						132	78							210
10					18	192	72							282
11					36	48								84
12					57	108	18							183
13					30	48	6							84
14				4	36	72								112
15					36	12								48
16					69	24								93
17				12	36									48
18					72									72
19					12	24								36
20				36	12									48
21				12										12
22				36	36									72
23				12										12
25					30									30
26				12										12
28				24	3									27
29					6									6
30					6									6
31				24										24
34				12										12
35				12										12
37			12											12
38			6	12										18
40			6	12										18
41			12											12
46			6											6
48			12											12
50			6											6
74			6											6
88		12												12
196	1													1
Total	1	12	66	220	495	792	924	792	495	220	66	12	1	4096

Number of restricted domains on which an SCF exists.

SCF
1: aaac-c
2: aaa-a-
3: bbbbbb
4: bbbbbb
5: cccccc
6: cccccc

Table 3: Parallel (im)possibilities.

Correspondence between number of SCFs and number of SWFs .

	#swf																
#scf	2	3	4	5	6	7	8	9	10	11	12	13	14	15	17	20	total
2	169																169
3	24	228															252
4	6	84	345														435
5		6	144	302													452
6		24	24	168	204												420
7				36	192	138											366
8			24	36	78	168	68										374
9				12	12	60	96	30									210
10				36	36	18	120	60	12								282
11						24	12	24	24								84
12					30	36	30	24	48	12	3						183
13			6			24	24		12	18							84
14						24	48		30	10							112
15							12	24		12							48
16					12		18	48	12		3						93
17								12	24			12					48
18									60	12							72
19					12	12			12								36
20								6	6	24	12						48
21											12						12
22							12	24			24	12					72
23												12					12
25								24	6								30
26											12						12
28									3	24							27
29								6									6
30							6										6
31											24						24
34											12						12
35											12						12
37													12				12
38										12				6			18
40										12			6				18
41														12			12
46													6				6
48													12				12
50													6				6
74													6				6
88															12		12
196																1	1
total	199	342	543	590	576	504	446	282	249	136	114	36	48	18	12	1	4096

Number of domains on which *j* SCFs and *k* SWFs co-exist.

Result 1. *The impossibility result no longer occurs if more than half of the 12 profiles have been eliminated both for SWF and SCF.*

Result 2. *The possibility may occur if more than two of the 12 profiles are eliminated appropriately both for SWF and SCF.*

Result 2 suggests that at least one profile for each cross-adjacent pairs in the two minimal super-Arrovian domains is necessary and sufficient for the parallel impossibility. However, this is not correct (see (3, 2) and (4, 2) in Table 3 and also an SCF between Table 2 and Table 3).

Result 3. *(i) There are 169 domains where Arrow-type aggregation (SWF) and non-dictatorial non-imposed strategy-proof voting (SCF) are both empty. (ii) There are also 30 domains where SCF exists but SWF is empty. (iii) There is no domain where SWF exists but SCF is empty. (iv) In the other domains, SWF and SCF are both non-empty.*

Additionally, if we substitute Maskin monotonicity and unanimity for strategy-proofness and non-imposition, then Table 2 is the same as shown in Table 1.

References

[1] Fishburn, P. C., Kelly, J. S. 1997. Super-Arrovian domains with strict preferences. *SIAM Journal on Discrete Mathematics*, 10(1), 83–95.

[2] Indo, K. 2007. Proving Arrow's theorem by Prolog, *Computational Economics,* 30(1), 57–63.

[3] Indo, K. 2009. Modeling a small agent society based on the social choice logic programming. In T. Terano et al. (Eds.), *Agent-based Approaches in Economic and Social Complex Systems V.* Springer Verlag.

[4] Indo, K. 2010. Generating social welfare functions over restricted domains for two individuals and three alternatives using Prolog, In A. Tavidze (Ed.), *Progress in Economics Research*, 18, Nova Science,.

Appendix The program used in the presented paper is available at http://p.tl/HqRL.

```
Programs:
%ranking
rc( 1, [a, c, b]).
rc( 2, [a, b, c]).
rc( 3, [b, a, c]).
%and so on.
```

```
%generic function form
f([ ], [ ], _).
f([ X - Y | F ], [ X | D ],
Axiom):-
   f(F, D, Axiom),
   G =.. [ Axiom, X, Y, F],
   G.
```

```
%axioms of social choice
swf_axiom( X, Y, F):- rc( _, Y),
   pareto( X - Y),
   iia( X - Y, F).
scf_axiom( X, Y, F):-  x( Y),
     ¥+ manipulable( _, X - Y, F).
```

```
%SWF/SCF
swf( F, D):- f( F, D, swf_axiom),
   ¥+ dictatorial_swf( _, F).
scf( F, D):- f( F, D, scf_axiom),
    non_imposed(F),
    ¥+ dictatorial_scf(_,F).
```